

UNIT - 3

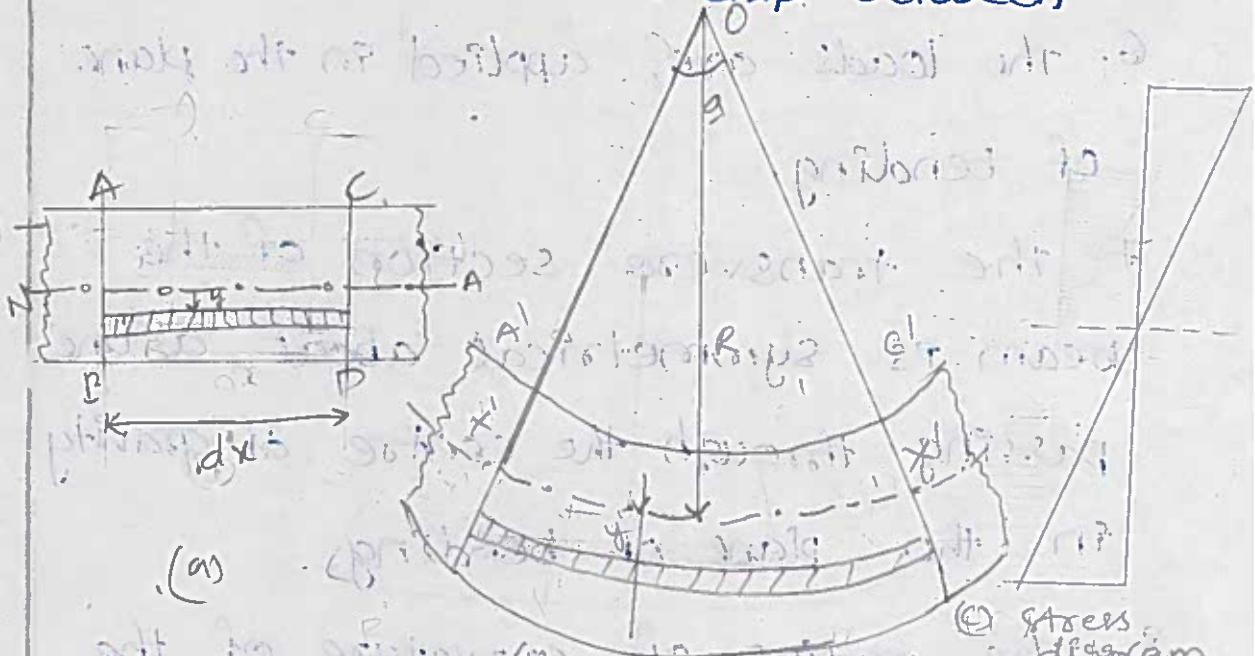
FLEXURAL STRESSES

[Ref:- Strength of Material - Dr. R.K. Bansal]

Theory of simple Bending :-

When a beam is loaded it is bent and subjected to bending moments.

Consequently longitudinal or bending stresses are induced in cross section in order to determine the practical utility of any beam. It is necessary to establish a relationship between



The radius of curvature of which the beam bends.

Assumptions Made In Theory of simple Bending :-

1. The material of the beam is perfectly homogeneous throughout
2. The stress induced is proportional to

to the strain and at no place the stress exceeds the elastic limit.

3. The value of modulus of elasticity (E) is same for the fibres of the beam under compression or under tension.
4. The transverse section of the beam, which is plane before bending, remains plane after bending.
5. There is no resultant pull or push or push on the cross section of the beam.
6. The loads are applied in the plane of bending.
7. The transverse section of the beam is symmetrical about a line passing through the centre of gravity in the plane of bending.
8. The radius of curvature of the beam before bending is very large in comparison to its transverse dimension.

Expression for Bending Stress:-

fig shows a longitudinal section of a beam, the neutral layer (axis)

(v) Bending bent to form an arc of circle of radius R . The neutral layer is then before bending, the length EF which after bending becomes $E'F'$

Consider some part of length δx will be determined as shown in fig. Let $A'B'$ and $C'D'$ meet at O .

Let R = Radius of neutral layer $N'N$

θ = angle subtended at (O) by $F'B'$ and $C'D'$ produced

Consider a layer EF at a distance y below the neutral layer NN . After bending this layer will be elongated to $E'F'$

Original length of layer $EF = \delta x$

Also, length of neutral layer $NN = \delta x$.

After bending, the length of neutral layer NN' will remain unchanged. But length of layer $E'F'$ will increase.

Hence

$$N'N' = NN = \delta x$$

Now from fig

$$N'N' = R \times \theta \quad (\because \text{Radius of } E'F')$$

$$\text{and.} \quad E'F' = (R+y) \times \theta = R + y$$

$$NN' = NN = \delta x$$

$$\text{Hence } \delta x = R \times \theta$$

∴ Increase in the length of the layer

$$EF$$

$$= E'F' - EF$$

$$= (R+y)\theta - R\theta \quad [\because EF = R\theta = \delta x]$$

$$= y\theta$$

∴ Strain in the layer EF

Increase in length

original length.

$$\frac{y\theta}{EF} = \frac{y\theta}{R\theta}$$

$$[\because EF = R\theta]$$

$$\therefore \frac{y}{R}$$

AS ' R ' is constant hence the strain in a layer is proportional to its distance from the neutral axis.

The above equation shows the variation of strain along the depth of the beam. The variation of strain is linear.

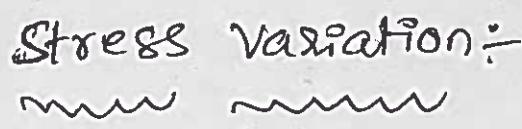
$$3x9 = 27$$

$$3x(0.9) = 2.7$$

$$\text{Unit-} \text{kg m/sq ft}$$

(3)

Stress Variation :-



$$\sigma \propto y =$$

$\sigma = \sigma_0$

$$\sigma \propto \frac{y}{R} =$$

let $\sigma =$ stress in the layer EF

~~27.01.2022 and 01.02.2022~~

~~Young's modulus of the beam~~

then $E =$

Stress in the layer EF

Strain in the layer EF

\therefore strain
in
 $EF = \frac{y}{R}$

$$E = \frac{\sigma}{\left(\frac{y}{R}\right)}$$

$$\sigma = E \times \left[\frac{y}{R}\right]$$

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}}$$

Position of Neutral Axis:-

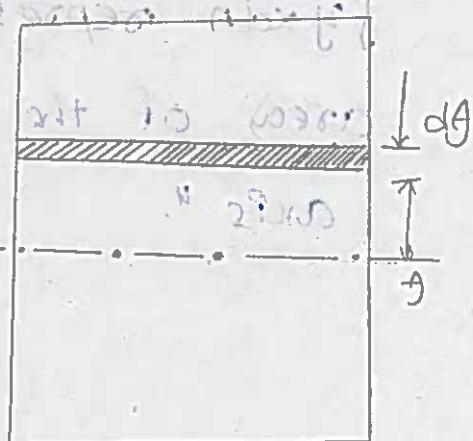
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If a transverse section of the beam is now considered let a strip of area ' dA ' lie at a distance ' y ' from the neutral axis.

let dA = area of the layer

Now the force on the layer

= Stress on layer \times Area of layer



$$= \sigma \times dA$$

$$= \frac{E}{R} \times y \times dA \quad [\because \sigma = \frac{E}{R} \times y]$$

Total force on the beam section is obtained by integrating the above equation

\therefore Total force on the beam section

$$= \int \frac{E}{R} \times y \times dA$$

$$= \frac{E}{R} \int y \times dA \quad [\because E, R \text{ constants}]$$

But for pure bending, there is no force on the section of the beam (or force is zero).

$$\therefore \frac{E}{R} \int y \times dA = 0$$

$$\boxed{\int y \times dA = 0}$$

Now ' $\int y \times dA$ ' represents the moment of area dA about 'Neutral axis'. Hence $\int y \times dA$ represents the moment of entire area of the section about "neutral axis".

(A)

Moment of Resistance:- Due to pure bending the layers above the N.A are subjected to compressive stresses whereas the layers below the N.A are subjected to tensile stresses. Due to these stresses, the forces will be acting on the layers. The force on the layer at a distance 'y' from neutral axis is by the equation.

$$\text{force on layer} = \frac{E}{R} xy \, dA$$

moment of this force about N.A

$$= \text{force on layer } xy$$

$$= \frac{E}{R} xy \, dA \, xy$$

$$= \frac{E}{R} xy^2 \, dA$$

Total moment of the forces on the section of the beam

$$= \int \frac{E}{R} xy^2 \, dA$$

$$= \frac{E}{R} \int y^2 \, dA$$

Let M = external moment applied on the beam section for equilibrium the moment of resistance offered by the section should be equal to the external bending moment.

$$\therefore M = \frac{E}{R} \int y^2 x dA$$

But the expression $\int y^2 x dA$ represents the moment of inertia of the area of the section about the neutral axis.

Let this moment of inertia be I .

$$\therefore \frac{M}{I} = \frac{E}{R} \quad \text{--- (1)}$$

We know

$$\frac{\sigma}{y} = \frac{E}{R} \quad \text{--- (2)}$$

From equation (1) & (2) we can write as

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

This is known as bending equation.

Section Modulus:

Referring to the bending equation

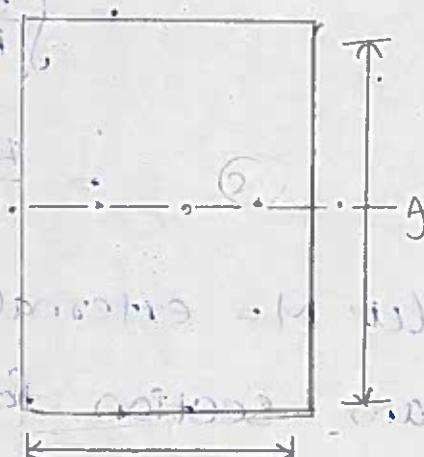
$$\frac{M}{I} = \frac{\sigma}{y}, \text{ we have}$$

$$\sigma = \frac{M \cdot y}{I}$$

$$\sigma = \frac{M}{\left(\frac{I}{y}\right)}$$

(or)

$$\sigma = \frac{M}{Z}$$



(3)

where \bar{z} = section modulus = $\frac{I}{y}$

The section modulus is usually quoted for all standard sections and practically is of greater use than the second moment of area (i.e. M.O.I.)

The strength of the beam section depends mainly on the section modulus.

The section modulus of rectangular and circular sections are calculated below.

Rectangular Section:-

Fig shows a rectangular section of width 'b' and depth 'd'. Let the horizontal centroidal axis be neutral axis section.

Section modulus (\bar{z}) = moment of Inertia about the N.A

i) Distance of the most distant point of the section from the neutral axis.

$$\bar{z} = \frac{I}{y_{\text{max}}}$$

$$\text{But } I = \frac{bd^3}{12}; \text{ and } y_{\text{max}} = \frac{d}{2}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}}$$

$$Z = \frac{bd^3}{6}$$

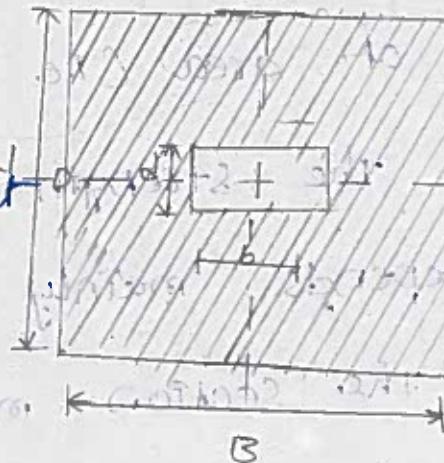
Hollow Rectangular section:-

Fig shows a rectangular hollow section.

moment of inertia about

N.A.

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$



$$I = \frac{1}{12} [BD^3 - bd^3]$$

$$y_{max} = \frac{D}{2}$$

$$\therefore \text{Section modulus } Z = \frac{I}{y_{max}}$$

$$Z = \frac{(BD^3 - bd^3)/12}{d/2} = \frac{BD^3 - bd^3}{6D}$$

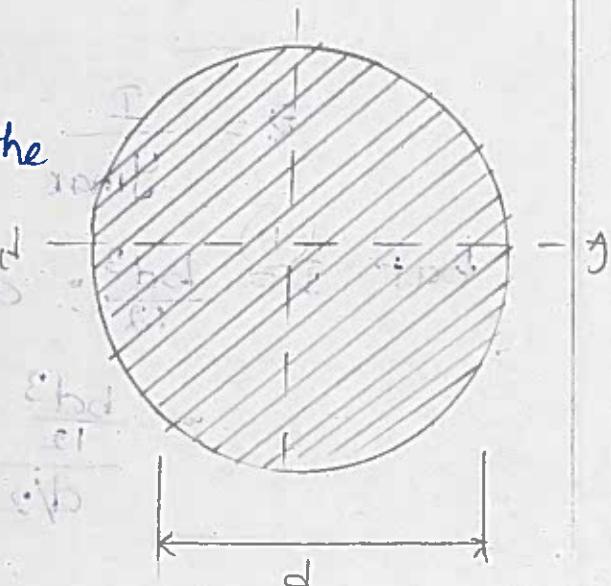
$$Z = \frac{BD^3 - bd^3}{6D}$$

iii) Circular section:-

moment of inertia of the

section, about the
neutral axis.

$$I = \frac{\pi d^4}{64}$$



$$⑥ y_{\max} = \frac{d}{2}$$

$$\therefore \text{Section modulus } Z = \frac{\frac{\pi}{4} r^3}{\text{max}}$$

$$Q = \frac{\pi d^4}{dH} \frac{1}{d/2}$$

$$Z = \frac{\pi d^4}{64} \times \frac{2}{d}$$

$$\therefore 2 = \frac{\pi d^3}{32}$$

iv) Hollow Circular Section:

Four wavy lines representing the boundary of the central island.

moment of inertia of the

Section about the Nettopia

QXPS

$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$y_{max} = \frac{D}{2}$$

\therefore Section modulus

$$2P = \frac{F}{y_{max}}$$

$$Z = \frac{\pi(D^4 - d^4)}{64} \times \frac{2}{D}$$

$$2 = \frac{\pi}{32} \left[-\frac{D^4 - d^4}{D} \right]$$

v) Triangular Section:-

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$$\frac{b}{2} \times \frac{h}{3}$$

cm<sup>2</sup>

moment of inertia of the

Section about the neutral  
axis

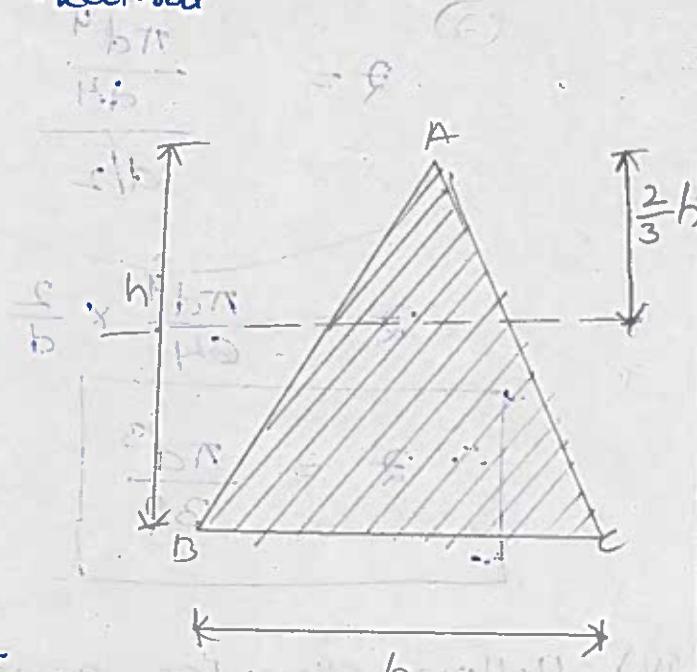
$$I = \frac{bb^3}{36}$$

$$y_{\text{max}} = \frac{2}{3} h$$

$$\therefore z = \frac{I}{y_{\text{max}}}$$

$$z = \frac{bb^3/36}{2/3 h}$$

$$\boxed{\therefore z = \frac{bb^2}{24}}$$



Shear Stress:-

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In the previous lesson we have

discussed the theory of simple bending.

In this theory, we assumed that no

shear force F_S

acting on the

section, But in

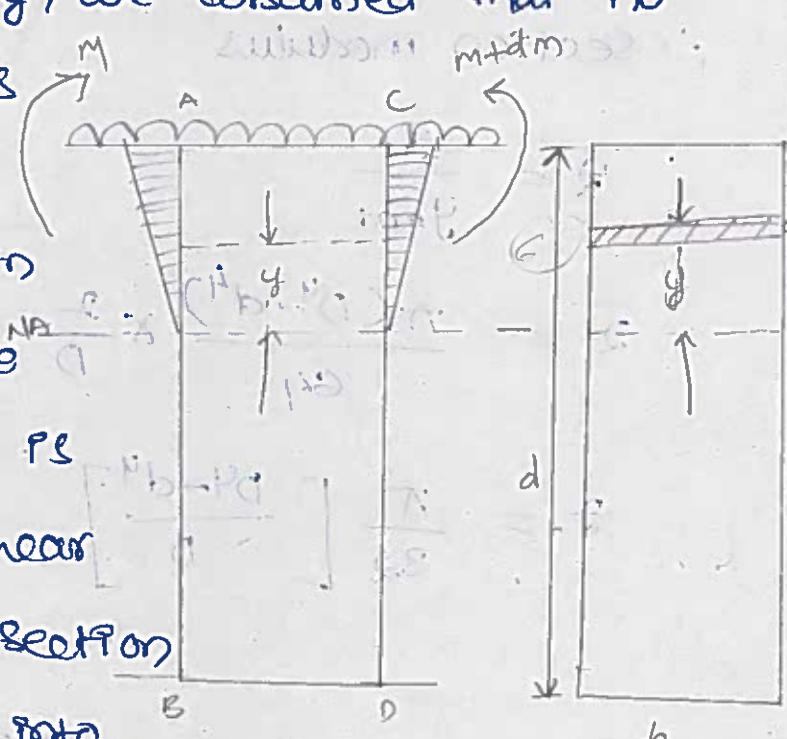
actual practice

when a beam is

loaded, the shear

force, at a section

always comes into



(2) comes into play, along with the bending moment. It has been observed that the effect of shearing stress, as compared to the bending stress, is quite negligible, and is not of much importance. But sometimes, the shearing stress at a section assumes much importance in the design criterion.

Derivation of formula:-

Consider a small portion ABCD of length dx of a beam loaded with uniformly distributed load as shown in fig.

We know that when a beam is loaded with a uniformly distributed load, the shear force and bending moment vary at every point along the length of the beam.

Let $M =$ Bending moment at AB.

$M + dM =$ Bending moment at CD,

$F =$ Shear force at AB

$F + dF =$ Shear force at CD, and

$I =$ moment of inertia of the section about N.A.

Now consider an elementary strip at a distance 'y' from the neutral axis as shown in fig.

Now let σ = intensity of bending stress across AB at distance y from the N.A
 dA = cross-section area of the strip

we have already discussed that

$$\frac{M}{I} = \frac{\sigma}{y} \text{ (or) } \sigma = \frac{M}{I} \times y$$

$$\text{Similarly, } \sigma + d\sigma = \frac{M + dm}{I} \times y$$

when $\sigma + d\sigma$ = intensity of bending stress across CD

we know that the force acting across

$$AB = \text{stress} \times \text{Area}$$

$$= \sigma \times dA$$

$$= \frac{M}{I} \times y \times dA \quad \text{--- (1)}$$

Similarly, force acting across

$$CD = (\sigma + d\sigma) \times dA$$

$$= \frac{M + dm}{I} \times y \times dA \quad \text{--- (2)}$$

(8)

∴ let unbalanced force on the strip

$$= \frac{M+dm}{I} xy \, dA - \frac{M}{I} xy \, dA$$

$$df = \text{force} = \frac{dm}{I} xy \, dA$$

∴ the total unbalance force (F) above the neutral axis may be found out by integrating the above equation between 0 and $d/2$

$$F = \int_0^{d/2} \frac{dm}{I} x(y \, dA)$$

$$= \frac{dm}{I} \int_0^{d/2} y \, dA \quad [\because \{y \, dA = A\bar{y}\}]$$

$$F = \frac{dm}{I} \cdot A\bar{y}$$

where A = Area of beam above neutral

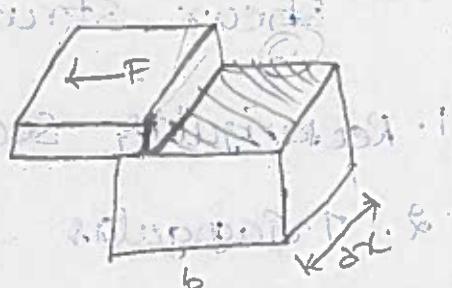
axis

\bar{y} = distance between the centre of gravity of the area and the neutral axis.

we know that the intensity of shear

stress.

$$\tau = \frac{\text{Total force}}{\text{Area}}$$



$$T' = \frac{\frac{dm}{dx} A\bar{y}}{b}$$

[∴ when b is the width of beam]

$$T' = \frac{du}{dx} \times \frac{A\bar{y}}{Ib}$$

$T' = F \times \frac{A\bar{y}}{Ib}$

[∴ substitute $\frac{du}{dx} = F$ = shear force]

Distribution of shearing stress:

we have obtained a relation which helps us in determining the value of shear stress at any section on a beam. for doing so, we shall calculate the intensity of shear stresses at important sections of a beam and then sketch a shear stress diagram.

such a diagram helps us in obtaining the value of shear stress at any section along the depth of the beam.

we shall discuss the distribution of shear stress over the following sections

1. Rectangular Sections

2. Triangular Section.

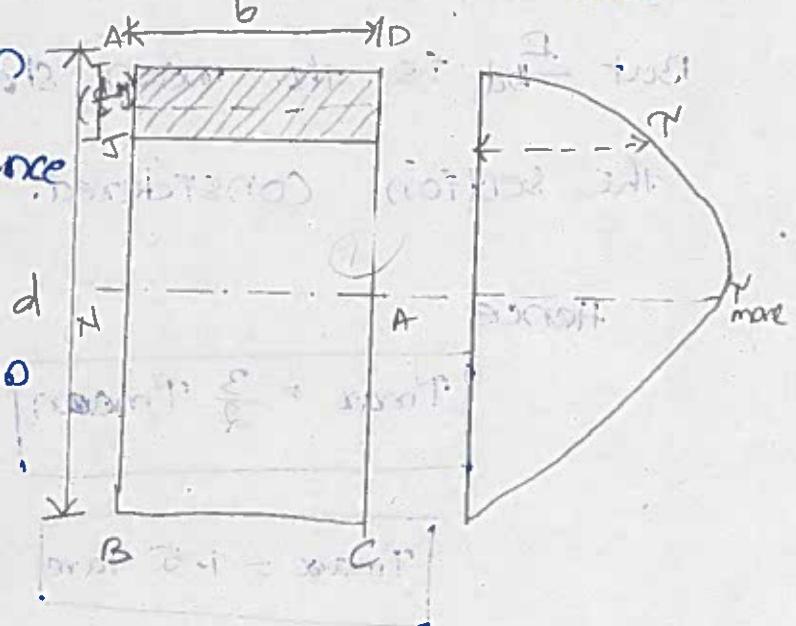
- 2. Triangular section
- 3. Circular section
- 4. I - sections
- 5. T - Sections and
- 6. Miscellaneous sections

F = shear force at section
 A = Area of section above(y)
 \bar{y} = Distance of the shaded area from the N.A.
 $A\bar{y}$ = moment of shaded area about the N.A.
 I = moment of inertia

Rectangular Section:

Let 'b' be the width and d be the depth of rectangular section let T be the shear stress in at layer at distance y from N.A. where

a particular section is subjected to shear force S.F



$$\text{Shaded area } A = b \left[\frac{d}{2} - y \right]$$

$$\therefore y = \frac{1}{2} \left[\frac{d}{2} + y \right] \Rightarrow y + \frac{1}{2} \left[\frac{d}{2} - y \right] = \frac{y}{2} + \frac{d}{2} = \frac{1}{2} \left[\frac{d}{2} + y \right]$$

$$I = \frac{bd^3}{12}$$

$$\therefore A\bar{y} = \frac{b}{2} \left[\frac{d}{2} - y \right] \left[\frac{d}{2} + y \right]$$

$$= \frac{b}{2} \left[\frac{d^2}{4} - y^2 \right]$$

$$\text{Now } \tau = \frac{F \cdot A \cdot y}{I \cdot b} = \frac{F \times \frac{b}{2} \left[\frac{d^3}{4} - y^2 \right]}{\frac{bd^3}{12} \times b}$$

$$\tau = \frac{12F \times b}{b \times bd^3 \times 2} \left[\frac{d^3}{4} - y^2 \right]$$

The maximum value of τ is at the neutral axis when $y=0$

$$\tau_{\max} = \frac{12F \cdot d^2}{bd^3 \times 8}$$

$$\tau_{\max} = \frac{3}{2} \times \frac{F}{bd}$$

$$\therefore \tau_{\text{ave}} = \frac{F}{bd}$$

But $\frac{F}{bd}$ is the mean shear stress (τ_{ave}) at the section considered.

Hence

$$\tau_{\max} = \frac{3}{2} \tau_{\text{mean}}$$

$$\tau_{\max} = 1.5 \tau_{\text{ave}}$$

Circular Section

Consider a circular section of diameter d as shown in fig we know that the shear stress on a layer at distance ' y ' from the neutral axis.

(10)

$$T = F \times \frac{Ag}{Ib}$$

$$p \cdot (p - 1) P = q b \sqrt{R^2 - y^2}$$

we know that in

a circular section

width of the strip

$$JK \Rightarrow b = 2\sqrt{R^2 - y^2}$$

and area of the

shaded strip.

$$dA = 2\sqrt{R^2 - y^2} dy$$

\therefore moment of this area about the neutral axis

$$dA \cdot y = 2y \sqrt{R^2 - y^2} dy$$

Now moment of the whole shaded area above the neutral axis may be found by integrating the above equation within the limit

y and R.

$$\therefore Ag = \int_y^R 2y \sqrt{R^2 - y^2} dy$$

$$[\text{Eq. 7.3. } A \bar{y} = \int_y^R b \cdot y dy] - ① \quad [\because b = 2\sqrt{R^2 - y^2}]$$

we know that width of the strip JK

$$b = 2\sqrt{R^2 - y^2}$$

$$b^2 = 4(R^2 - y^2)$$

differentiating both sides of the above equation

$$2b \cdot db = 4(-2y) dy$$

$$= -8y \cdot dy$$

$$y \cdot dy = -\frac{1}{4} b \cdot db$$

substituting the value of $y \cdot dy$ in

equation ①

$$A\bar{y} = \int_y^r b \cdot \left(-\frac{1}{4} b \cdot db\right)$$

$$A\bar{y} = -\frac{1}{4} \int_y^r b^2 db - ②$$

we know that when $y=y$; width $b=b$,

and when $y=R$, width $b=0$;

therefore the limits of integration,

may be changed from ' y ' to R from

' b ' to zero in equation ②.

$$A\bar{y} = -\frac{1}{4} \int_b^0 b^2 db$$

$$A\bar{y} = \frac{1}{4} \int_0^b b^2 db \quad [\because \text{eliminating } -\text{ve sign}]$$

$$A\bar{y} = \frac{1}{4} \left[\frac{b^3}{3} \right]_0^b$$

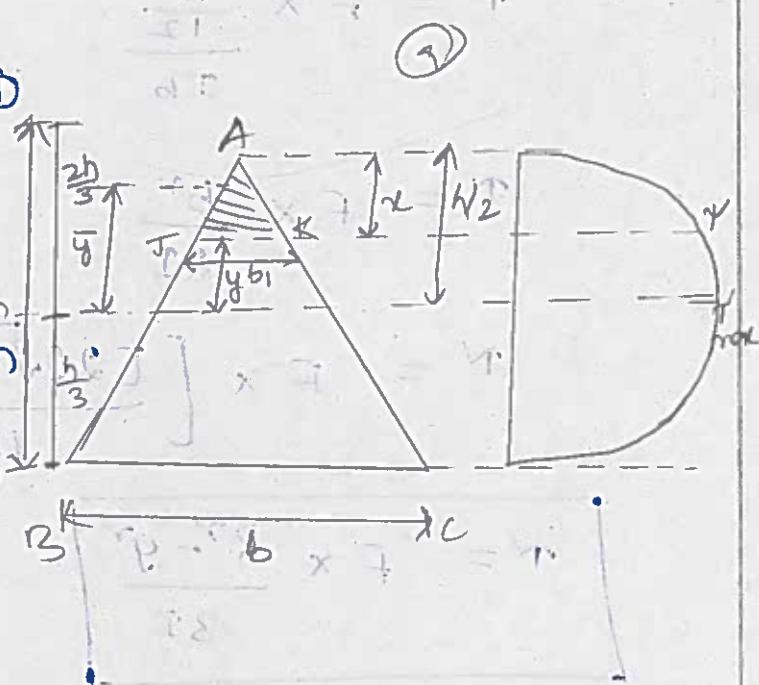
$$A\bar{y} = \frac{b^3}{12}$$

Triangular section :-

considers a beam of triangular cross-section ABC of base b and height (h) as shown in fig.

$$\tau = F \times \frac{A\bar{y}}{2b} \quad \text{--- ①}$$

We know that the shear stress is on a layer JK at a distance 'y' from the N.A.



we know that width of the strip JK.

$$b_x = \frac{bx}{h}$$

∴ Area of the shaded portion AJK.

$$A = \frac{1}{2} \times JK \times h$$

$$A = \frac{1}{2} \left[\frac{bx}{h} \cdot x \right]$$

$$A = \frac{bx^2}{2h}$$

$$\text{and } \bar{y} = \frac{2h}{3} - \frac{2x}{3} = \frac{2}{3}(h-x)$$

Substitute the values of b, A and \bar{y} in equation ①

$$\tau = \frac{F \times \left[\frac{bx^2}{2h} \right] \times \frac{2}{3}(h-x)}{I \times \frac{bx}{h}}$$

(ii) Now substituting this value of $A \cdot y$ in our original formula for the shear stress c.c

$$\tau = F \times \frac{Ay}{Ib}$$

$$\tau = F \times \frac{\frac{b^3}{12}}{Ib}$$

$$\tau = F \times \frac{b^2}{12I}$$

$$\tau = F \times \left[\frac{[2(\sqrt{R^2 - y^2})]^2}{12I} \right]$$

$$\boxed{\tau = F \times \frac{R^2 - y^2}{3I}}$$

Thus we again see that τ increases as y decrease at a point where $y = r$; $\tau = 0$; and when y is zero τ is maximum.

\therefore substituting $y = 0$; and $I = \frac{\pi}{64} d^4$

$$\therefore \tau_{\max} = F \times \frac{\frac{R^2 - 0}{3 \times \frac{\pi}{64} d^4}}{\frac{3 \times \frac{\pi}{64} d^4}{3 \times \frac{\pi}{4} d^4}} = \frac{4F \left(\frac{d}{2} \right)^2}{3 \times \frac{\pi}{4} d^4}$$

$$= \frac{4F}{3 \times \frac{\pi}{4} d^2}$$

$$\boxed{\tau_{\max} = 1.33 \tau_{\text{avg}}}$$

$$\gamma = \frac{F}{3I} \times [x(h-x)]$$

$$\boxed{\gamma = \frac{F}{3I} [hx - x^2]} \quad -\textcircled{2}$$

thus we see that the variation of γ with respect to x is parabola. we also see that at a point where

$$x=0, x=h; \gamma=0 \text{ at N.A where } x = \frac{2h}{3}$$

$$\gamma = \frac{F}{3I} \left[h \times \frac{2h}{3} - \left(\frac{2h}{3} \right)^2 \right]$$

$$= \frac{F}{3I} \times \frac{2h^2}{9}$$

$$\gamma = \frac{2Fh^2}{27I}$$

$$= \frac{2Fh^2}{27 \times \frac{bh^3}{36}} \quad [\because I = \frac{bh^3}{36}]$$

$$\gamma = \frac{8F}{3bh} \quad [\because \text{Area} = \frac{bh}{2}]$$

$$\gamma = \frac{4}{3} \frac{F}{\text{Area}}$$

$$\boxed{\gamma = 1.33 \frac{F}{\text{Area}}}$$

Now for maximum intensity, differentiating $\textcircled{2}$ and equating to zero

$$\frac{d\gamma}{dx} \left[\frac{F}{3I} (hx - x^2) \right] = 0$$

$$h - 2x = 9 \text{ (or) } x = \frac{h}{2}$$

Now substituting this value of x in equation ①.

$$P_{\max} = \frac{F}{3I} \left[h \times \frac{b}{2} - \left(\frac{b}{2} \right)^2 \right]$$

$$= \frac{fb^2}{12T}$$

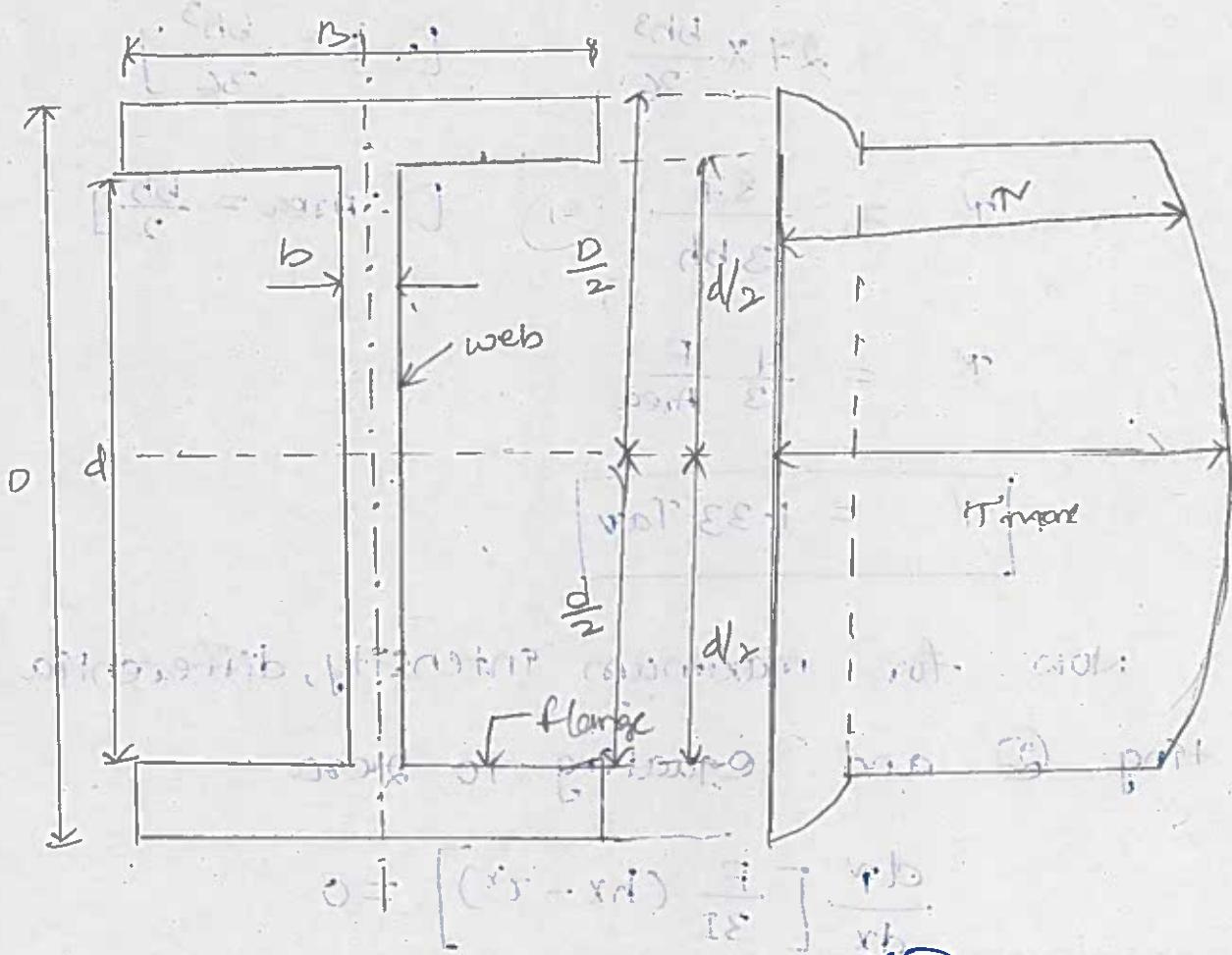
$$= \frac{Fh^2}{12 \times \frac{bh^3}{36}}$$

$$= \frac{3F}{bh} = \frac{3}{2} \times \frac{F}{\text{Area}}$$

$$T_{\text{max}} = 1.5 \times T_{\text{av}}$$

$$[\because T_{av} = \frac{F}{\text{Area}}]$$

Symmetrical I-section:-



(3) When y is greater than $d/2$:

Shearing Stress in Hinges:

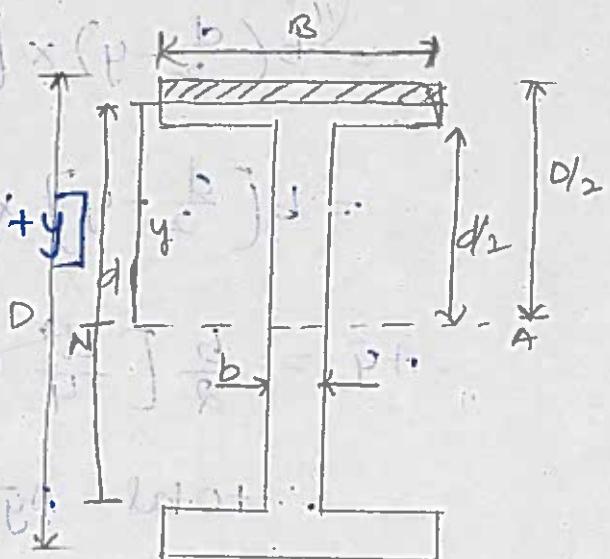
$$\tau = \frac{F}{Ib} \times A\bar{y}$$

$$A\bar{y} = B \left[\frac{D}{2} - y \right] \times \frac{1}{2} \left[\frac{D}{2} + y \right]$$

$$= \frac{B}{2} \left(\frac{D^2}{4} - y^2 \right)$$

$$\tau = \frac{F}{Ib} \times \frac{B}{2} \left(\frac{D^2}{4} - y^2 \right)$$

$$\boxed{\tau = \frac{F}{2I} \left[\frac{D^2}{4} - y^2 \right]}$$



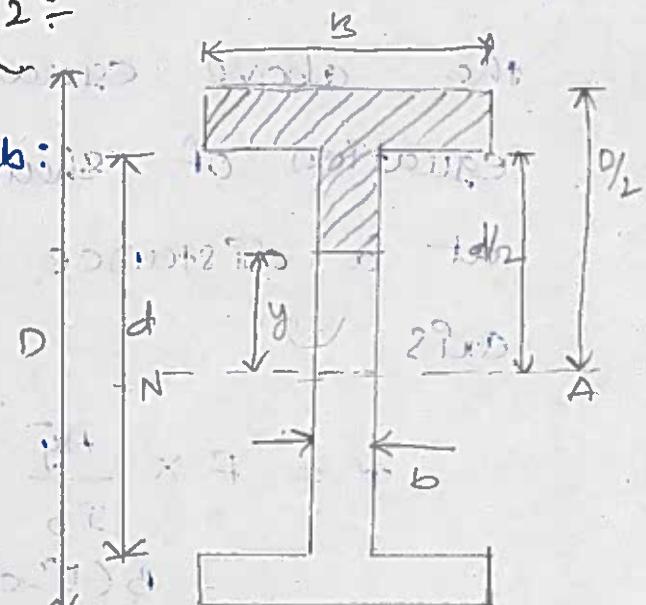
$$\text{when } y = D/2 = 0$$

$$y = d/2 = \frac{F}{8I} (D^2 - d^2)$$

ii) when y is less than $y/2$:

shearing stress in web:

$$\tau = \frac{F \cdot A\bar{y}}{Ib}$$



It means that y lies in the web as shown in fig.

In this case, the value of $A\bar{y}$ for the flange

$$\therefore A\bar{y} = B \left[\frac{D}{2} - \frac{d}{2} \right] \times \left[\frac{d}{2} + \frac{1}{2} \left[\frac{D}{2} - \frac{d}{2} \right] \right]$$

$$= B \left[\frac{D-d}{2} \right] \left[\frac{1}{2} \left(\frac{D+d}{2} \right) \right]$$

$$\boxed{A\bar{y} = \frac{B(D^2 - d^2)}{8}} \quad \text{--- (1)}$$

② and the value of $A\bar{y}$ for the web above N.A

$$= b \left(\frac{d}{2} - y \right) \times \left[y + \frac{1}{2} \left(\frac{d}{2} - y \right) \right]$$

$$= b \left(\frac{d}{2} - y \right) \times \left[\frac{1}{2} \left(\frac{d}{2} + y \right) \right]$$

$$A\bar{y} = \frac{b}{2} \left[\frac{d^2}{4} - y^2 \right] \quad \text{②}$$

$$\therefore \text{total } A\bar{y} = \text{①} + \text{②}$$

$$A\bar{y} = \frac{B(D^2 - d^2)}{8} + \frac{b}{2} \left[\frac{d^2}{4} - y^2 \right]$$

Now substituting the value of $A\bar{y}$ from the above equation. In our original equation of shear stresses on a layer at a distance 'y' from the neutral axis.

$$\tau = F \times \frac{A\bar{y}}{I_b}$$

$$= F \times \frac{\frac{B(D^2 - d^2)}{8} + \frac{b}{2} \left[\frac{d^2}{4} - y^2 \right]}{I_b}$$

$$= \frac{F}{I_b} \left[\frac{B(D^2 - d^2)}{8} + \frac{b}{2} \left(\frac{d^2}{4} - y^2 \right) \right]$$

at N.A when $y = 0$; shear stress is maximum

$$\therefore \tau_{\max} = \frac{F}{I_b} \left[\frac{B}{8} (D^2 - d^2) + \frac{b}{2} \left(\frac{d^2}{4} - 0 \right) \right]$$

$$= \frac{F}{Ib} \left[\frac{B}{8} (D^r - d^r) + \frac{bd^r}{8} \right]$$

Now, shear stress at the junction of the top of the web and bottom of the flange

$$\begin{aligned}\tau_{\max} &= \frac{F}{Ib} \left[\frac{B}{8} (D^r - d^r) \right] + \frac{b}{2} \left(\frac{d^r}{4} - j^r \right) \\ &= \frac{F}{Ib} \left[\frac{B}{8} (D^r - d^r) + \frac{b}{8} \left(\frac{d^r}{4} - \frac{d^r}{4} \right) \right] \\ \tau_{\max} &= \frac{F}{Ib} \left[\frac{B}{8} (D^r - d^r) \right]\end{aligned}$$

$\left[\because \text{sub } y = \frac{d}{2} \right]$

Note:-

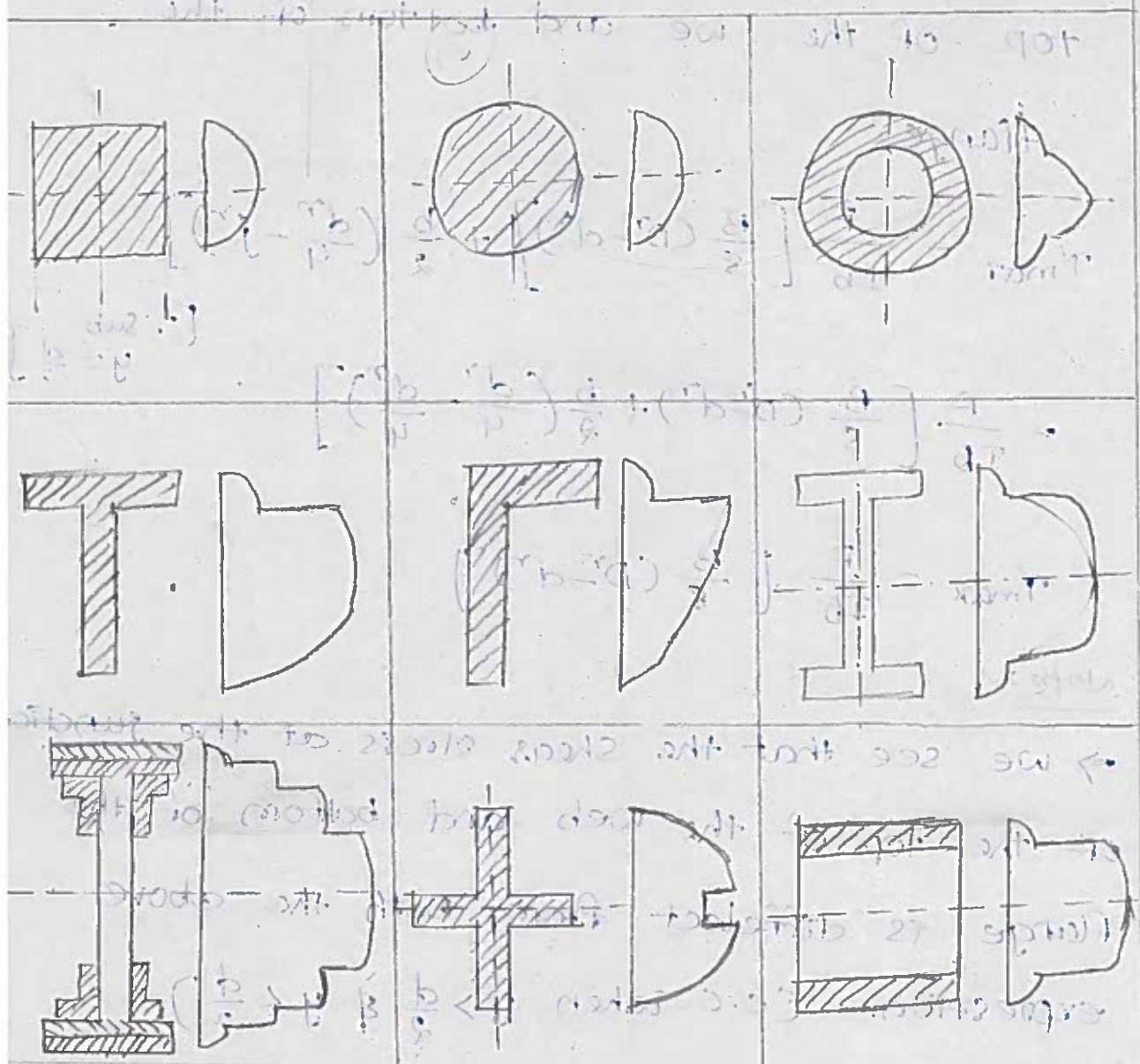
→ we see that the shear stress at the junction of the top of the web and bottom of the flange is different from both the above expression. (C.C when $y > \frac{d}{2}$ & $y < \frac{d}{2}$)

→ we also see that the shear stress changes absolutely from

$$\frac{F}{OI} (D^r - d^r) \text{ to } \frac{F}{SI} \cdot \frac{B}{b} (D^r - d^r)$$

thus the shear stress at this junction increases by $(\frac{B}{b})$ times as shown in fig.

Shear stress distribution for typical sections:



cross section properties

$$(b, r) \frac{d}{4} \text{ or } (b, r) \frac{r}{12}$$

where b is width and r is radius of semi-circle.

for more details see page 200-201